Metal Pipe Tuning

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Finding the Formula for Pipe Length

When tuning a metal pipe which is to be *struck* — so that the pipe itself resonates, rather than the air inside — the relationship between the frequency of the pipe and the length is the following:

Relationship between Frequency and Struck Pipe Length

The frequency of a struck pipe is inversely proportional to the square of its length:

$$f = \frac{K}{l^2}$$

If we solve for l, we end up with this formula for the length of a pipe based on K and f:

Calculating pipe length based on frequency and K:

$$l = \sqrt{\frac{K}{f}}$$

The value of K is related to the properties of the metal and is quite complicated to calculate (it involves the elastic properties of the metal, scary-looking equations and calculus). But, we can obtain our own K value by measuring the length (l) and frequency (f) of one pipe, and then solving for K. We'll call this the *calibration constant*. We can use any length of pipe and it doesn't matter what frequency we end up with, we just need to measure accurately.

Finding the Calibration Constant:

$$K = f * l^2$$

Example: If we measure a length of 24 inches, and a frequency of 386 Hz, our K would be:

$$K = 386 * 24^2 = 222336$$

Assuming our pipe material is consistent, we can plug the calibration constant and desired frequency into the length formula to get the length for any frequency.

Finding the Frequency of Any Note

In order to find the frequency of any note in the chromatic scale, we need to know the *ratio of an equal-tempered half step* which is *the twelfth root of two*. This is because we need to divide an octave (a ratio of 2) into 12 equal smaller *ratios* - such that a frequency multiplied by this ratio 12 times is doubled - in other words, r multiplied by itself 12 times equals 2:

The Ratio of an Equal-Tempered Half Step

$$r^{12} = 2, \text{ therefore } r = \sqrt[12]{2} \approx 1.05946$$

Multiplying any frequency by this number provides a frequency one half step above the starting frequency; *dividing* any frequency by this number provides a frequency one half step below the starting frequency. *Repeated* multiplying or dividing can give us a frequency any number of half steps away from a starting frequency, so multiplying a frequency by this number 12 times will double the frequency, and dividing 12 times will halve the frequency. *If we use A440 as our starting frequency, we can find any note in the chromatic scale*.

Note: raising a number to a positive power implies repeated multiplication; raising a number to a negative power implies repeated division.

Frequency f_{note} based on distance d in half steps from A440:

$$f_{note} = 440 * (\sqrt[12]{2})^{d}$$

Fun Fact: You can divide an octave into an arbitrary number of n equal parts by taking the nth root of 2! So the ratio for a quarter tone (dividing the octave into 24 parts) is $\sqrt[24]{2}$, and the ratio for 1 cent (one 100th of a semitone, which divides the octave into 1200 parts) is $\sqrt[1200]{2}$!

Examples: If we wanted to know the frequency of the Bb one half step above A440 ($d = 1$):

$$f_{note} = 440 * (\sqrt[12]{2})^{1} \approx 466.164$$

To find the frequency of Middle C, 9 half steps *below* A440 ($d = -9$):

$$f_{note} = 440 * (\sqrt[12]{2})^{-9} \approx 261.626$$

Note that you can use any starting frequency. The frequencies of A's are nice round numbers and so work very well. See the table below for the frequencies of A's that can be used as starting points for calculating the frequencies of notes:

Frequencies of A's spanning the range of human hearing (20 Hz – 20,000 Hz):

A0: 27.5	A1: 55	A2: 110	A3: 220	A4: 440
A5: 880	A6: 1760	A7: 3520	A8: 7040	A9: 14080

We can also find Middle C by measuring 3 half steps *above* the A220 ($d = 3$):

$$f_{note} = 220 * (\sqrt[12]{2})^{3} \approx 261.626$$

Measuring and Cutting the Pipe

Remember that the length of a pipe with frequency f_{note} can be calculated like so:

$$l = \sqrt{\frac{K}{f_{note}}}$$

Therefore the length of a pipe with a K value of 222336 and a frequency of 261.626 Hz (Middle C) would be:

Length of a pipe with $K = 222336$ tuned to Middle C ($f_{note} = 261.626$):

$$l = \sqrt{\frac{K}{f_{note}}} = \sqrt{\frac{222336}{261.626}} \approx \sqrt{849.824} \approx 29.15 \text{ inches}$$

Converting to 16ths of an Inch

Note that the fractional part of this length is in 100ths of an inch (0.15) rather than 16ths of an inch, which is what your tape measure provides. To convert the fractional part to 16ths, multiply it by 16:

$$16\text{ths of an inch} = \text{fractional portion of length} * 16$$

$$0.15 * 16 = \frac{2.4}{16}$$

$$\text{Therefore:}$$

$$29.15 \text{ inches} = 29\frac{2.4}{16} \text{ inches}$$

Tips

- Remember to re-calibrate (find a new value of K) with each new pipe that you intend on cutting into lengths.
- Measure twice, cut once! Don't cut the pipe unless you are *sure* you are cutting the right length.
- Recommended pipe material:
  - An inexpensive option is EMT conduit, which is thin and tends to be inconsistent, meaning the math doesn't always work out perfectly after calibrating if you are making big changes to the pipe length.
  - When using EMT conduit, I have found that it is best to cut the pipe around an inch longer than it should be, and then *re-calibrate* with that shorter pipe. Then you can get a more accurate measurement and cut the pipe once more to zero in on the desired frequency.
  - Third Coast Percussion recommends 3/4" galvanized steel conduit, which is thicker and more consistent than EMT conduit, but is also more expensive.
- Rotary pipe cutters are fairly inexpensive ($20-$30) and easy to use - you don't need to use power tools to cut the pipes!

Here you can access an interactive pipe tuning utility that will help you find the calibration constant and then provide the length of pipe for any chromatic note you choose:
www.brianmuellercreations.com/pipes/